

(i) intensities measured quantitatively and with sufficient accuracy are needed for a reliable determination of a polytype unless its period is quite small (systematic errors have to be allowed for);

(ii) a method is needed which gives all polytypes in reasonable agreement with the observed data.

We believe that our method meets this requirement. These views are confirmed by Gomes de Mesquita (1969) who successfully applied our method to the determination of a 120R polytype of SiC. In this case the R value obtained was 6.6% and was further reduced to 5.9% by allowing for secondary extinction. For comparison, the observed and calculated intensities obtained by Tokonami (1966) in his test example of a 96R polytype are given in Table 10, with the intensities again arranged in order of decreasing observed values.

Table 10. *Intensity values observed and calculated by Tokonami (1966) for his test example, arranged in order of decreasing I_{obs} values*

l	I_{obs}	I_{calc}	l	I_{obs}	I_{calc}
22	128	22	28	16	8
31	96	60	82	16	6
79	96	72	88	16	1

Table 10 (cont.)

l	I_{obs}	I_{calc}	l	I_{obs}	I_{calc}
46	64	31	40	8	11
64	64	289	43	8	0
49	64	256	67	8	0.6
34	64	126	73	8	0.4
37	64	36	85	8	0
16	64	27	91	8	1
70	64	0	1	0	0.1
10	32	5	4	0	0.2
25	32	19	52	0	12
67	32	4	55	0	0.9
19	32	21	58	0	0.1
7	16	2	61	0	0.7
13	16	11	94	0	0.7

We would like to express our thanks to Mr P. Kovács and Mr E. Lendvay who kindly supplied the crystals used in our experiments.

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Complex Lattice Potentials in Electron Diffraction Calculated for a Number of Crystals

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Structure potentials, V_g , and absorption potentials for 100 keV-electrons are given in tabulated form for almost all monatomic crystals with elements $Z=3$ to 90 and for a number of crystals of the rock-salt type. The absorption potentials are given in the form of the Fourier coefficients C_{0g}^i of the Yoshioka imaginary potential for excitation of crystal electrons and excitation of phonons at the temperatures 20, 93 and 293°K. All computations are based on numerical Hartree-Fock-Slater atomic radial functions by Herman & Skillman (*Atomic Structure Calculations*, Englewood Cliffs: Prentice Hall, 1963). The calculations show that the ratio $C_{0g}^i(\text{el})/V_g$, where $C_{0g}^i(\text{el})$ refers to excitation of single crystal electrons, lies between 0.005 and 0.012 for the lower reflexion vectors g and practically all Z . In contrast to this the ratio $C_{0g}^i(\text{phonon})/V_g$ is much larger and increases about linearly with Z , for a given g .

1. Introduction

The scattering of fast electrons in crystals can be understood in terms of a complex periodic potential by which the electrons are supposed to be diffracted. Its real part consists of the electrical potential of the crystal atoms plus their dynamical polarization induced by the passing external electrons, while the imaginary part represents the inelastic scattering suffered by these electrons during passage (Yoshioka, 1957).

Calculations of the imaginary crystal potentials have been done on the basis of the Thomas-Fermi atomic model for MgO (Yoshioka, 1957) and later, for Li, Be, Ne, Al and Cu by use of Hartree-Fock atomic functions (Whelan, 1956a). In these two papers the excitation of the tightly bound crystal electrons was dealt with. This mechanism, however, proved to be too weak to fully account for the experimental anomalous absorption effects, and it was soon found that the thermal diffuse scattering (phonon excitation) is the

main contribution to the total imaginary potential. Imaginary potentials due to phonon excitation have been calculated by Yoshioka & Kainuma (1962) and by Whelan (1965*b*) for several crystals, and, on the basis of a different theoretical approach by Hall & Hirsch (1965), and using the same theory by Humphreys & Hirsch (1968). All these computations, as far as they involve atomic numbers $Z > 28$, rely on Thomas-Fermi-Dirac electron scattering factors. Meanwhile new Hartree-Fock-Slater (HFS) atomic radial functions have become available for all elements of the periodic system. These functions are needed for the calculation of both the real and imaginary lattice potentials. On the other hand more experimental results have been reported which should be compared with theory.

In the present paper a table of the complex lattice potentials is given which has been computed with the use of the HFS atomic functions by Herman & Skillman (1963). The evaluation of the imaginary potentials has been shortened by an analytical solution of an angle integration previously done numerically or substituted by a mean value. The Table contains the imaginary potentials due to the excitation of core electrons, plasma electrons and phonons, together with the structure potentials, for a number of crystals. The dependences on the reflexion vector \mathbf{g} , the temperature and the atomic weight are discussed.

2. Complex lattice potential

On transmission through a perfect crystal the high energy electrons can be described as being diffracted by the crystal potential

$$V(\mathbf{r}) = \sum_{\mathbf{g}} (V_{\mathbf{g}} + iV_{\mathbf{g}}^i) \exp(-i\mathbf{g} \cdot \mathbf{r}). \quad (1)$$

The sum is over all reciprocal lattice vectors \mathbf{g} . $V_{\mathbf{g}}$ is the usual structure potential. $V_{\mathbf{g}}^i$ represents the imaginary potential which accounts for the inelastic scattering. The most important mechanisms of inelastic scattering are the excitation of phonons, the excitation of plasmons and of (tightly bound) core electrons. Each one of these mechanisms contributes a part to $V_{\mathbf{g}}^i$ denoted by $C_{0\mathbf{g}}^i$. Other processes of minor importance including the production of Bremsstrahlung are omitted.

Virtual inelastic scattering gives rise to a small addition to the real potential (Yoshioka, 1957) which will be ignored. In the following a number of formulae are compiled which are relevant for the understanding of the Tables.

3. Structure potentials $V_{\mathbf{g}}$

The Fourier coefficient of the crystal potential is given by

$$V_{\mathbf{g}} = - \frac{e^2}{\epsilon_0 V_c g^2} \sum_{\alpha} [Z^{(\alpha)} - f^{(\alpha)}(\mathbf{g})] \exp(-i\mathbf{g} \cdot \mathbf{R}_{\alpha}) \quad (2)$$

$f^{(\alpha)}(\mathbf{g})$ is the atomic X-ray scattering amplitude for the atom located at \mathbf{R}_{α} ($\alpha=1, \dots, r$) in the unit cell of volume V_c . $f(\mathbf{s}) = \sum_j f_j(\mathbf{s})$ is a sum over the subshells of

the atom. The functions $f_j^{(\alpha)}(\mathbf{s})$ have been calculated from the tabulated atomic radial wave functions published by Herman & Skillman (1963).

Comments on the evaluation of V_0

As is well known, the value of the mean inner potential V_0 depends much on the outer electrons of the crystal atoms.

One way of calculating V_0 would be to assume the crystal to consist of free neutral atoms. Another extreme way is to take ions to be at the lattice points and the remaining electrons to be distributed uniformly. The contribution of p perfectly free electrons to V_0 is (Bethe, 1928):

$$V_0 = - \frac{3}{10} \frac{e^2 p}{4\pi\epsilon_0 r_0}; \quad (3)$$

r_0 is chosen so that a sphere of radius r_0 has the same volume as the crystal atom.

In the table two values of V_0 are given for each elementary crystal which may be considered as the upper and lower limits of the actual values. The upper value listed as the first number in the row designated by $V_{\mathbf{g}}$ is based on the assumption of free neutral atoms in the crystal. The lower limit given in the (000)-line and $V_{\mathbf{g}}$ -row has been calculated in the following way: V_0 of the p -times ionized free atom (*i.e.* the p outermost electrons of the neutral atom are missing) plus the V_0 of p free electrons from equation (3). This procedure is different from, and believed to be more appropriate than, taking *free* ions plus p free electrons.

4. Imaginary structure potentials $C_{0\mathbf{g}}^i$

The imaginary structure potentials incorporate the absorption properties of the crystal with respect to the incident electrons. They therefore depend on their energy, $E_0 = \hbar^2 k_0^2 / 2m$.

4.1. Excitation of electrons

Calculating the absorption due to excitation of core electrons gives (Yoshioka, 1957; see also Whelan, 1965*a*)

$$C_{0\mathbf{g}}^i(\text{core}) = - \frac{\hbar^2}{2m} \frac{4}{a_0^{*2} k_0 V_c} \int \frac{S(\mathbf{s}, \mathbf{s} - \mathbf{g})}{s^2 (\mathbf{s} - \mathbf{g})^2} d\sigma, \quad (4)$$

where

$$S(\mathbf{s}, \mathbf{s} - \mathbf{g}) = \sum_{\alpha} \exp(-i\mathbf{g} \cdot \mathbf{R}_{\alpha}) (f^{(\alpha)}(\mathbf{g}) - \sum_j f_j^{(\alpha)}(\mathbf{s}) f_j^{(\alpha)}(\mathbf{s} - \mathbf{g}))$$

$a_0^* = a_0 m_0 / m$ is the relativistic Bohr radius.

An exchange term in $S(\mathbf{s}, \mathbf{s} - \mathbf{g})$ which arises from overlapping of the subshells in an atom has been ignored in the course of the derivation of this formula as has any overlapping of neighbouring crystal atoms. Furthermore, this formula must be applied only to

closed-shell structures of atoms. This restriction, however, is not severe since the inner electrons contribute almost nothing to the value of $C_{0g}^i(\text{core})$, in contrast to C_{00}^i where this question will be considered in more detail (§4.2). We therefore include the non-closed outer shells in the sum of equation (5) by assuming them to be spherically symmetric, which also leads to $S(\mathbf{s}, \mathbf{s}-\mathbf{g}) = S(s, |\mathbf{s}-\mathbf{g}|)$.

The coordinate system used for the integration of the absorption potentials is shown in Fig. 1. In this coordinate system expression (4) reads

$$C_{0g}^i(\text{core}) = -\frac{\hbar^2}{2m} \frac{4}{a_0^* k_0 V_c} \sum_{\alpha} \exp(-ig\mathbf{R}_{\alpha}) \times \int_0^{2k_0} \int_0^{2\pi} \frac{[f^{(\omega)}(2a) - \sum_i f_j^{(\omega)}(\sqrt{a^2+r^2+2ar \cos \varphi}) f_j^{(\omega)}(\sqrt{a^2+r^2-2ar \cos \varphi})] r dr d\varphi}{(a^2+r^2)^2 - (2ar \cos \varphi)^2} \quad (6)$$

$$= -\frac{\hbar^2}{2m} \frac{4}{a_0^* k_0 V_c} \sum_{\alpha} \exp(-ig\mathbf{R}_{\alpha}) \times \int_0^{2k_0} \frac{f^{(\omega)}(2a) - \sum_j f_j^{(\omega)}(\sqrt{|a^2-r^2|}) f^{(\omega)}(\sqrt{2(a^2+r^2)-|a^2-r^2|})}{(a^2+r^2)|a^2-r^2|} r dr. \quad (7)$$

The φ -integration has been performed with the help of the integration procedure given in the Appendix.

The imaginary potential due to crystal electron excitation is then given by

$$C_{0g}^i(\text{el}) = C_{0g}^i(\text{core}) + C_{0g}^i(\text{plasmon}).$$

In addition collective and single-electron excitation give rise to a another contribution to C_{0g}^i which is of

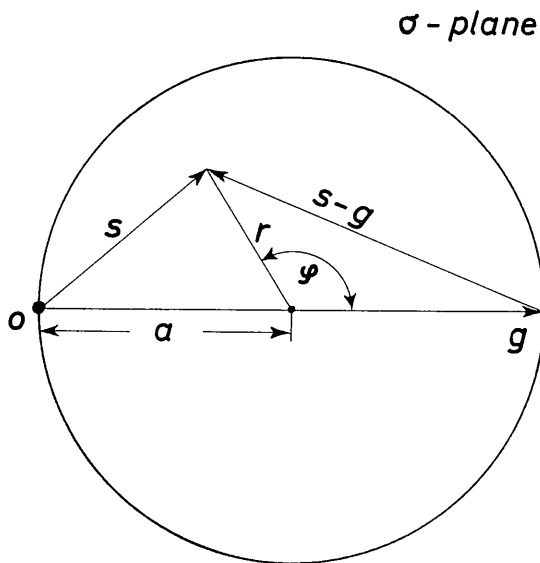


Fig. 1. Coordinate system used for integration of C_{0g}^i potentials. \mathbf{s} , scattering vector, \mathbf{g} , reciprocal lattice vector.

opposite sign. These contributions have not been calculated here (see Radi, 1968).

For $\mathbf{g}=0$ the electronic absorption consists of the core part alone, as will be seen in the next section.

4.2. Plasmon excitation

In the case of plasmon excitation (Ferrell, 1956):

$$C_{00}^i(\text{plasmon}) = -\frac{\hbar\omega_p}{2a_0 k_0} \ln \frac{s_c}{s_{\min}} \quad (8)$$

and (Radi, 1968)

$$C_{0g}^i(\text{plasmon}) = 0 \quad (\mathbf{g} \neq 0). \quad (9)$$

For $C_{00}^i(\text{plasmon})$ the problem is to determine the number, p , of free electrons per atom taking part in collective oscillations. This is easily done in cases where the crystal absorption spectrum shows a definite plasmon peak which is in agreement with

$$\omega_p = \sqrt{\frac{e^2 n}{m_0 \epsilon_0}}, \quad (10)$$

where $n = pA(0)/V_c$, the density of the plasma electrons, and p is a natural number; $A(0)$ = number of atoms in the unit cell. In the other cases, p had to be chosen by arguments of chemical valence, *i.e.* p = the number of electrons in the outmost subshell. Although this is somewhat arbitrary, the consequences are not too critical in our application to the estimation of the mean electronic absorption.

Introducing $s_c \simeq \omega_p/v_F$ as the maximum wave number of a plasmon, with $v_F = (\hbar/m_0)\sqrt{3\pi^2 n} = \hbar k_F/m_0$, the Fermi velocity, and $s_{\min} = \omega_p m/(\hbar k_0)$, we can rewrite (8):

$$C_{00}^i(\text{plasmon}) = -\frac{\hbar\omega_p}{2a_0 k_0} \ln \frac{k_0}{k_F(1 + E_0/m_0 c^2)}. \quad (11)$$

With p given, this C_{00}^i is determined entirely. p , $\hbar\omega_p$ and $C_{00}^i(\text{plasmon})$ are listed in the $C_{0g}^i(\text{el})$ rows of the Tables.

Since $C_{00}^i(\text{core})$ depends on $s_{\min} = \overline{\Delta E}m/(\hbar^2 k_0)$ one must choose a realistic $\overline{\Delta E}$, which ought to be done by consideration of the experimental energy losses for core electron excitation. We shall use a universal $s_{\min} = 0.02 \text{ \AA}^{-1}$ instead, because this value corresponds to the usual aperture size and only electrons scattered into

Table 1. Monatomic crystals. Theoretical structure potentials V_g and Yoshioka absorption potentials for 100 keV-electrons due to the excitation of crystal electrons C_g^i(el) and excitation of phonons C_g^i(phonon) at T=20°K, 93°K and 293°K

V_g^i in the last row represents the total imaginary potential coefficient at 293°K, being the sum of the third and the sixth row
For every particular lattice type (dia, b.c.c., f.c.c., h.c.p.) a set of reflections, g, have been chosen (indices g_i in the first row)
The V_0 value in the (000, V_g) spot represents a lower limit and the number immediately above this represents an upper limit for the mean inner potential V_0. On top of the C_g^i (el) row, C_00^i (plasmon) is shown separately. This is calculated on the basis of p free electrons per atom. The number p is indicated above the third row together with the plasmon energy EP. Above the rows designated by C_g^i (phonon) the absolute temperature and mean-square vibration amplitude, u_g^2 (denoted by U2), pertaining to the three rows are given. DT is the Debye temperature of the crystal. DT=300** means a dummy number, used in cases where no DT was available. The corresponding absorption potentials are irrelevant.

All real and imaginary lattice potentials are given in eV and with opposite signs as they are usually negative quantities. Absorption potentials at electron energies other than 100 keV may be obtained with the help of the conversion factors given in Table 3

DIA A=3.56 P=4 EP=31.3 DT=2020 T=20 93 293 U2=0.0013 .0017 .0020 C 6

DIA A=5.42 P=4 EP=16.6 DT=580 T=20 93 293 U2=0.0020 .0024 .0045 SI 14

DIA A=5.66 P=4 EP=15.6 DT=306 T=20 93 293 U2=0.0019 .003A .0091 GF 3P

DIA A=6.49 P=4 EP=12.7 DT=260 T=20 93 293 U2=0.0008 .0016 .0049 SN 50

BCC A=3.46 P=1 EP=8.2 DT=316 T=20 93 293 U2=0.0182 .0247 .0562 LI 3

BCC A=4.30 P=1 EP=5.9 DT=163 T=20 93 293 U2=0.0108 .0254 .0726 NA 11

BCC A=5.33 P=1 EP=4.3 DT=132 T=20 93 293 U2=0.0080 .0230 .0705 K 19

BCC A=3.03 P=2 EP=14.1 DT=356 T=20 93 293 U2=0.0021 .0031 .0075 V 23

BCC A=2.89 P=1 EP=10.7 DT=445 T=20 93 293 U2=0.001A .0020 .0045 CR 24

Table 1 (cont.)

MCP		ZR 41		MCP		TC 43		MCP		RU 44						
A=3.23 P=2		UT=269		A=2.73 P=2		DT=300**		A=2.71 P=1		DT=413						
A3=5.14 EP=10.9		T=20 93 293		A3=4.39 EP=13.9		T=20 93 293		A3=4.28 EP=10.1		T=20 93 293						
U2=.0015 .0025 .0067		CIG(EL) CIG(PHONON) VIG		U2=.0014 .0019 .0049		CIG(EL) CIG(PHONON) VIG		U2=.0008 .0011 .0025		CIG(EL) CIG(PHONON) VIG						
G	VG	CIG(EL)	CIG(PHONON)	VIG	G	VG	CIG(EL)	CIG(PHONON)	VIG	G	VG	CIG(EL)	CIG(PHONON)	VIG		
000	15.49	2.516	.283	.386	.662	3.18				000	24.30	4.680	.382	.472	.785	5.44
002	10.58	.088	.287	.389	.651	74				002	16.09	.130	.376	.463	.761	.89
100	5.69	.051	.138	.188	.319	.37				100	8.61	.079	.191	.236	.388	.47
101	8.80	.070	.246	.333	.558	.63				101	13.49	.102	.323	.398	.653	.75
102	3.97	.024	.138	.186	.308	.33				102	6.15	.031	.189	.232	.378	.41
110	6.72	.034	.277	.372	.605	.64				110	10.43	.039	.362	.444	.718	.76
112	5.57	.023	.264	.353	.566	.59				112	8.54	.024	.358	.438	.699	.72
201	4.75	.019	.228	.304	.487	.50				201	7.28	.020	.314	.384	.612	.63
104	2.30	.008	.128	.170	.266	.27				104	3.45	.008	.173	.211	.332	.34
203	3.61	.012	.223	.294	.451	.46				203	5.40	.012	.297	.361	.563	.57
114	3.78	.012	.250	.329	.496	.51				114	5.58	.012	.338	.410	.632	.64
211	3.40	.011	.219	.288	.438	.45				211	5.07	.011	.295	.359	.556	.57
006	3.02	.009	.241	.313	.449	.46				006	4.37	.010	.324	.390	.586	.59
123	2.81	.008	.211	.276	.402	.41				123	4.11	.009	.285	.344	.521	.53
205	2.50	.007	.206	.277	.379	.38				205	3.61	.009	.276	.332	.495	.50
220	2.78	.008	.235	.304	.427	.43				220	4.03	.010	.317	.380	.565	.57
132	1.23	.004	.115	.147	.199	.20				132	1.77	.005	.154	.185	.269	.27

MCP		CU 48		MCP		LA 57		MCP		CE 58						
A=2.98 P=2		DT=146		A=3.75 P=2		DT=142		A=3.65 P=2		DT=140						
A3=5.62 EP=11.3		T=20 93 293		A3=6.06 EP=8.6		T=20 93 293		A3=5.96 EP=9.0		T=20 93 293						
U2=.0027 .0066 .0200		CIG(EL) CIG(PHONON) VIG		U2=.0018 .0044 .0145		CIG(EL) CIG(PHONON) VIG		U2=.0022 .0052 .0154		CIG(EL) CIG(PHONON) VIG						
G	VG	CIG(EL)	CIG(PHONON)	VIG	G	VG	CIG(EL)	CIG(PHONON)	VIG	G	VG	CIG(EL)	CIG(PHONON)	VIG		
000	15.92	2.278	.604	.976	1.613	3.89				000	14.32	2.217	.484	.778	1.309	3.53
002	12.49	.153	.590	.946	1.514	1.66				002	10.72	.111	.477	.761	1.247	1.35
100	5.92	.066	.306	.485	.761	.82				100	5.61	.061	.242	.385	.631	.69
101	9.63	.096	.518	.820	1.282	1.37				101	8.82	.087	.408	.651	1.066	1.15
102	4.75	.036	.293	.460	.698	.73				102	4.13	.031	.235	.371	.592	.62
110	7.67	.039	.571	.877	1.252	1.28				110	7.06	.043	.465	.727	1.132	1.17
112	6.76	.027	.553	.839	1.149	1.17				112	6.01	.030	.467	.720	1.077	1.10
201	5.58	.020	.476	.717	.962	.98				201	5.10	.025	.401	.619	.923	.94
104	3.15	.011	.273	.410	.546	.55				104	2.57	.011	.222	.339	.493	.50
203	4.58	.012	.458	.667	.816	.82				203	4.02	.015	.381	.576	.811	.82
114	5.04	.012	.519	.751	.897	.90				114	4.27	.015	.436	.653	.892	.90
211	4.03	.009	.442	.650	.717	.72				211	3.76	.014	.373	.561	.775	.79
006	4.49	.009	.504	.713	.796	.80				006	3.53	.011	.418	.611	.775	.78
123	3.49	.006	.426	.587	.608	.61				123	3.18	.010	.365	.535	.691	.70
205	3.39	.006	.420	.576	.585	.59				205	2.89	.008	.358	.518	.640	.65
220	3.23	.006	.457	.596	.517	.52				220	3.15	.009	.409	.586	.704	.71
132	1.45	.003	.218	.276	.215	.22				132	1.41	.004	.198	.278	.314	.32

MCP		PR 59		MCP		ND 60		MCP		GD 64						
A=3.66 P=2		DT=300**		A=3.65 P=2		DT=300**		A=3.63 P=2		DT=152						
A3=5.91 EP=9.0		T=20 93 293		A3=5.89 EP=9.0		T=20 93 293		A3=5.78 EP=9.1		T=20 93 293						
U2=.0010 .0014 .0035		CIG(EL) CIG(PHONON) VIG		U2=.0010 .0013 .0034		CIG(EL) CIG(PHONON) VIG		U2=.0015 .0032 .0103		CIG(EL) CIG(PHONON) VIG						
G	VG	CIG(EL)	CIG(PHONON)	VIG	G	VG	CIG(EL)	CIG(PHONON)	VIG	G	VG	CIG(EL)	CIG(PHONON)	VIG		
000	14.20	2.178	.304	.372	.643	2.82				000	14.81	2.391	.474	.736	1.317	3.71
002	10.62	.113	.301	.367	.631	.74				002	10.84	.123	.466	.720	1.269	1.39
100	5.60	.063	.153	.186	.320	.38				100	5.77	.070	.238	.367	.645	.71
101	8.81	.089	.258	.314	.541	.63				101	9.08	.099	.400	.618	1.089	1.19
102	4.13	.032	.149	.182	.310	.34				102	4.26	.037	.232	.355	.613	.65
110	7.11	.046	.296	.361	.612	.66				110	7.44	.054	.460	.703	1.193	1.24
112	6.06	.032	.299	.364	.611	.64				112	6.36	.037	.464	.703	1.157	1.19
201	5.16	.026	.258	.313	.526	.55				201	5.43	.031	.400	.606	.996	1.02
104	2.57	.011	.143	.173	.289	.30				104	2.70	.013	.223	.335	.538	.55
203	4.06	.016	.247	.299	.495	.51				203	4.31	.019	.381	.570	.900	.92
114	4.30	.016	.284	.343	.563	.58				114	4.57	.018	.447	.663	1.015	1.03
211	3.82	.014	.243	.294	.484	.50				211	4.09	.017	.375	.560	.874	.89
006	3.54	.011	.276	.331	.533	.54				006	3.76	.013	.425	.620	.900	.91
123	3.22	.010	.240	.289	.468	.48				123	3.46	.012	.369	.542	.804	.81
205	2.92	.009	.237	.285	.455	.46				205	3.12	.010	.362	.527	.754	.76
220	3.22	.009	.273	.328	.521	.53				220	3.44	.011	.418	.606	.856	.86
132	1.44	.004	.133	.160	.250	.25				132	1.57	.004	.205	.293	.395	.40

constituents of the compound (of Na-Cl type) to compute \bar{u}_g^2 from equation (15). The Debye temperature is taken from *International Tables* (1962).

5. Discussion

For the purpose of the discussion of some general aspects of the complex lattice potentials in Tables 1

and 2 it is useful to divide the absorption potentials, V_g^I , by the structure potential V_g , whereby the structure effects (lattice type) and the dependence on the degree of close packing, *i.e.* on the value of the lattice constants, are cancelled out. Furthermore, only these ratios, V_g^I/V_g and V_g^I/V_0^I , enter the formulae of electron diffraction theory to any great extent.

COMPLEX LATTICE POTENTIALS IN ELECTRON DIFFRACTION

Table 1 (cont.)

HCP			DY 66			HO 67		
A	P	Z	T	DT	DT	T	DT	DT
3.50	2	2	20	93	300**	20	93	300**
3.70	EP=9.3		U ₂ =0.0009	0.012	0.0030	U ₂ =0.0009	0.012	0.0030
			CIG(EPHONON)			CIG(EPHONON)		
G	VG	CIG(EL)	G	VG	CIG(EL)	G	VG	CIG(EL)
000	19.43	246	000	19.37	247	000	19.40	248
002	10.63	2.058	002	10.61	2.040	002	10.65	2.032
100	5.66	1.130	100	5.67	1.132	100	5.70	1.135
101	8.94	0.174	101	8.95	0.175	101	9.00	0.177
102	4.24	0.106	102	4.25	0.108	102	4.27	0.111
110	7.46	0.339	110	7.51	0.41	110	7.57	0.42
112	6.46	0.058	112	6.46	0.061	112	6.53	0.063
201	5.49	0.041	201	5.54	0.042	201	5.60	0.044
203	2.73	0.034	203	2.75	0.036	203	2.78	0.037
104	4.38	0.173	104	4.42	0.175	104	4.48	0.178
114	4.65	0.021	114	4.69	0.021	114	4.75	0.022
211	4.18	0.293	211	4.23	0.291	211	4.29	0.292
006	3.83	0.19	006	3.87	0.19	006	3.92	0.19
123	3.54	0.13	123	3.59	0.14	123	3.64	0.14
205	3.19	0.13	205	3.23	0.13	205	3.28	0.13
220	3.58	0.11	220	3.64	0.11	220	3.70	0.11
132	1.61	0.005	132	1.64	0.005	132	1.66	0.005

V_g itself represents the elastic scattering power of crystal in scattering electrons into the reflection g ; depends on the lattice constant and on the crystal structure of the atoms comprising the atomic lattice (small a) have larger structure factor V_g (see Tables and Fig. 2) decreases with increasing g . The main point is,

however, that for a given g , $C'_{0g}(e)/V_g$ does not vary noticeably with Z , being between 0.005 and 0.012 for practically all crystals in the lower range of reflections g . For the lower angle reflections, which experimentally are the only important ones, the ratio does depend somewhat on the atom's place in the periodic system, as for alkali and rare gas crystals the values are highest (max. 0.016 for Ne-111), but it does not vary systemati-

Table 2. Compound crystals (rock-salt type). Theoretical structure potentials, V_g , and Yoshioka absorption potentials, for 100 keV electrons, due to the excitation of crystal electrons $C_g^i(\text{el})$ and excitation of phonons $C_g^i(\text{phonon})$ at $T=20^\circ\text{K}$, 93°K and 293°K

V_g^i in the last row represents the total imaginary potential coefficient at 293°K being the sum of the third and the sixth row. Above the rows designated by C_g^i (phonon) the absolute temperature and mean-square vibration amplitude, u_s^2 (denoted by U2), pertaining to the three rows are given.

All real and imaginary lattice potentials are given in eV and with opposite signs, as they usually are negative quantities.

Absorption potentials and coefficients at electron energies other than 100 keV may be obtained with the help of the conversion factors given in Table 3.

3 LI +1 LIF					
9 F -1			A= 4.02 DT= 650		
T= 20 U2=.0050			T= 93 U2=.0052		
93 293			.0088		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	13.61	1.2n2	.076	.077	.105
111	-3.87	-.041	-.069	-.070	-.095
200	4.29	.044	.071	.073	.098
220	3.26	.019	.067	.068	.089
311	-2.15	-.006	-.059	-.060	-.077
222	2.63	.010	.062	.063	.080
400	2.20	.007	.057	.058	.072
331	-1.46	-.001	-.050	-.051	-.062
422	1.65	.003	.049	.050	.058
440	1.32	.002	.043	.043	.048
620	1.10	.001	.037	.038	.039
444	.94	.001	.033	.033	.032
642	.82	.000	.029	.029	.027
733	-.47	.000	-.023	-.023	-.019
662	.62	.000	.021	.021	.017
664	.54	.000	.018	.018	.013
933	-.32	.000	-.014	-.014	-.009

3 LI +1 LICL					
17 CL -1			A= 5.14 DT= 463		
T= 20 U2=.0043			T= 93 U2=.0048		
93 293			.0100		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	10.82	1.654	.094	.100	.153
111	-4.95	-.040	-.089	-.095	-.144
200	4.86	.036	.091	.098	.145
220	3.55	.013	.086	.091	.133
311	-2.61	-.003	-.081	-.086	-.123
222	2.78	.007	.082	.087	.123
400	2.29	.005	.078	.083	.114
331	-1.73	-.002	-.073	-.077	-.104
422	1.69	.004	.071	.075	.098
440	1.35	.004	.066	.069	.085
620	1.13	.003	.061	.063	.075
444	.98	.003	.057	.059	.066
642	.86	.003	.053	.055	.058
733	-.60	-.002	-.047	-.049	-.048
662	.68	.002	.046	.046	.044
664	.61	.002	.042	.042	.037
933	-.43	-.001	-.037	-.037	-.030

3 LI +1 LIBR					
35 BR -1			A= 5.49 DT= 387		
T= 20 U2=.0025			T= 93 U2=.0030		
93 293			.0069		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	11.40	1.980	.184	.205	.323
111	-6.09	-.068	-.182	-.203	-.316
200	5.89	.058	.179	.199	.311
220	4.44	.028	.179	.198	.303
311	-3.49	-.016	-.171	-.189	-.288
222	3.60	.019	.177	.195	.293
400	3.04	.014	.169	.186	.277
331	-2.50	-.010	-.166	-.183	-.268
422	2.37	.010	.164	.181	.259
440	1.97	.008	.155	.170	.237
620	1.71	.006	.150	.164	.221
444	1.51	.005	.144	.156	.205
642	1.36	.004	.138	.149	.190
733	-1.07	-.002	-.131	-.141	-.171
662	1.11	.002	.127	.135	.154
664	1.00	.002	.120	.127	.143
933	-.81	.000	-.113	-.119	-.129

3 LI +1 LII					
53 I -1			A= 6.00 DT= 331		
T= 20 U2=.0019			T= 93 U2=.0025		
93 293			.0060		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	12.12	2.432	.250	.292	.479
111	-7.33	-.084	-.245	-.285	-.466
200	6.95	.071	.247	.288	.469
220	5.30	.034	.242	.282	.452
311	-4.31	-.021	-.239	-.277	-.442
222	4.33	.023	.238	.275	.437
400	3.69	.017	.238	.275	.430
331	-3.15	-.012	-.231	-.267	-.413
422	2.91	.011	.228	.262	.399
440	2.44	.008	.222	.255	.378
620	2.12	.006	.215	.245	.354
444	1.88	.005	.209	.238	.336
642	1.69	.004	.201	.228	.314
733	-1.37	-.002	-.194	-.218	-.290
662	1.36	.002	.189	.211	.273
664	1.21	.002	.183	.203	.253
933	-1.02	-.001	-.175	-.194	-.233

11 NA +1 NAF					
9 F -1			A= 4.61 DT= 439		
T= 20 U2=.0046			T= 93 U2=.0052		
93 293			.0112		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	11.34	1.100	.083	.090	.136
111	-1.20	-.006	-.098	-.099	-.016
200	4.55	.062	.081	.087	.130
220	3.71	.031	.078	.083	.120
311	-1.52	-.002	-.096	-.097	-.011
222	3.14	.018	.074	.079	.110
400	2.71	.011	.071	.075	.102
331	-2.28	.001	-.094	-.095	-.008
422	2.13	.005	.064	.067	.086
440	1.75	.003	.058	.061	.072
620	1.48	.002	.053	.055	.061
444	1.28	.001	.048	.050	.051
642	1.13	.000	.044	.045	.043
733	-.03	.000	.000	.000	.000
662	.86	.000	.035	.036	.028
664	.75	.000	.031	.031	.022
933	-.01	.000	.000	.000	.000

11 NA +1 NACL					
17 CL -1			A= 5.63 DT= 281		
T= 20 U2=.0051			T= 93 U2=.0073		
93 293			.0190		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	9.54	1.430	.101	.123	.202
111	-3.08	-.018	-.055	-.068	-.113
200	4.81	.052	.098	.119	.188
220	3.75	.024	.094	.113	.172
311	-1.52	.004	-.049	-.059	-.090
222	3.08	.014	.091	.109	.158
400	2.62	.009	.087	.102	.143
331	-.92	.002	-.045	-.053	-.072
422	2.02	.006	.080	.093	.119
440	1.65	.004	.074	.085	.099
620	1.40	.004	.069	.078	.083
444	1.22	.003	.065	.071	.070
642	1.09	.003	.061	.066	.059
733	-.24	-.001	-.028	-.030	-.023
662	.86	.002	.052	.054	.039
664	.76	.002	.048	.048	.030
933	-.17	-.001	-.023	-.022	-.011

11 NA +1 NABR					
35 BR -1			A= 5.95 DT= 200		
T= 20 U2=.0041			T= 93 U2=.0074		
93 293			.0210		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	10.06	1.698	.208	.285	.459
111	-4.24	-.043	-.172	-.234	-.369
200	5.61	.070	.205	.278	.433
220	4.43	.036	.199	.268	.402
311	-2.37	-.008	-.164	-.219	-.318
222	3.70	.023	.194	.258	.372
400	3.19	.018	.192	.251	.347
331	-1.64	-.005	-.156	-.203	-.271
422	2.54	.012	.181	.232	.316
440	2.14	.009	.173	.217	.256
620	1.86	.007	.165	.202	.219
444	1.65	.005	.158	.190	.189
642	1.44	.004	.151	.178	.163
733	-.68	-.002	-.119	-.135	-.109
662	1.21	.003	.135	.150	.111
664	1.09	.002	.127	.137	.088
933	-.52	-.001	-.100	-.105	-.059

11 NA +1 NAI					
53 I -1			A= 6.46 DT= 198		
T= 20 U2=.0028			T= 93 U2=.0052		
93 293			.0147		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	10.56	2.058	.265	.369	.614
111	-5.51	-.061	-.242	-.335	-.550
200	6.41	.080	.265	.366	.596
220	5.07	.041	.260	.356	.567
311	-3.22	-.013	-.237	-.322	-.502
222	4.24	.027	.254	.346	.538
400	3.68	.020	.255	.344	.518
331	-2.31	-.008	-.228	-.306	-.453
422	2.95	.013	.240	.320	.462
440	2.50	.009	.233	.306	.419
620	2.18	.007	.224	.291	.378
444	1.94	.005	.217	.277	.343
642	1.76	.004	.212	.267	.312
733	-.99	-.002	-.186	-.230	-.249
662	1.42	.002	.196	.238	.244
664	1.28	.002	.188	.222	.211
933	-.74	-.001	-.165	-.192	-.169

19 K +1 K F					
9 F -1			A= 5.33 DT= 333		
T= 20 U2=.0044			T= 93 U2=.0056		
93 293			.0137		
G	VG	CIG(EL)	CIG(PHONON)	VIG	
000	11.38	1.242	.103	.119	.192
111	1.26	.012	.045	.051	.077
200	5.10	.062	.100	.115	.181
220	4.14	.027	.097	.112	.170
311	.80	.001	.042	.047	.068
222	3.47	.015	.094	.107	.158
400	2.97	.009	.090	.102	.145
331	.55	-.001	.039	.043	.057
422	2.30	.005	.084	.094	.125
440	1.87	.003	.077	.086	.106
620	1.57	.003	.072	.079	.091
444	1.35	.002	.067	.073	.079
642	1.19	.002	.063	.068	.068
733	.19	.001	.027	.028	.025
662	.92	.002	.054	.056	.047
664	.81	.002	.050	.051	.038
933	.15	.001	.022	.022	.015

Table 2 (cont.)

19 K +1 17 CL -1		A = 6.28 DT = 218 T = 20 U2 = .0052 .0088 .0244	K CL		19 K +1 35 BR -1		A = 6.58 DT = 152 T = 20 U2 = .0048 .0106 .0313	K BR		19 K +1 51 I -1		A = 7.05 DT = 158 T = 20 U2 = .0033 .0070 .0207	K I							
G	VG	CIG(EL)		CIG(PHONON)	VIG	G	VG	CIG(EL)		CIG(PHONON)	VIG	G	VG	CIG(EL)		CIG(PHONON)	VIG			
000	9.33	1.352	.107	.143	.239	1.59	000	9.57	1.536	.195	.295	.469	2.00	000	9.86	1.812	.239	.362	.597	2.41
111	-1.00	-.001	-.007	-.010	-.021	-.02	111	-2.09	-.023	-.112	-.165	-.250	-.27	111	-3.41	-.041	-.186	-.278	-.446	-.49
200	4.97	.059	.105	.140	.225	.28	200	5.51	.074	.190	.285	.438	.51	200	6.14	.081	.235	.353	.572	.65
220	4.01	.025	.102	.134	.206	.23	220	4.49	.036	.190	.279	.407	.44	220	4.99	.041	.239	.353	.552	.59
311	-1.38	.002	-.005	-.008	-.014	-.01	311	-1.15	-.007	-.106	-.152	-.210	-.22	311	-2.00	-.011	-.177	-.261	-.398	-.41
222	3.35	.013	.098	.128	.189	.20	222	3.80	.052	.185	.268	.373	.39	222	4.23	.026	.227	.336	.515	.54
400	2.87	.008	.096	.123	.173	.18	400	3.29	.015	.181	.257	.341	.36	400	3.69	.018	.225	.329	.488	.51
331	-.18	.000	-.004	-.006	-.010	.00	331	-.80	-.006	-.103	-.143	-.179	-.18	331	-1.45	-.008	-.178	-.254	-.363	-.37
422	2.22	.004	.089	.112	.144	.15	422	2.61	.009	.171	.236	.285	.29	422	2.96	.011	.219	.314	.438	.45
440	1.80	.003	.089	.113	.120	.12	440	2.18	.007	.163	.218	.238	.24	440	2.49	.008	.212	.298	.392	.40
620	1.52	.003	.079	.094	.100	.10	620	1.87	.006	.157	.204	.200	.21	620	2.16	.006	.208	.286	.353	.36
444	1.31	.003	.074	.087	.084	.09	444	1.65	.005	.150	.189	.168	.17	444	1.91	.005	.198	.269	.313	.32
642	1.15	.002	.070	.080	.071	.07	642	1.48	.004	.145	.176	.141	.14	642	1.72	.004	.193	.256	.281	.28
733	-.02	.000	-.002	-.002	-.002	.00	733	-.39	-.002	-.080	-.094	-.065	-.07	733	-.69	-.002	-.145	-.187	-.188	-.19
662	.90	.002	.062	.065	.046	.05	662	1.19	.003	.131	.147	.090	.09	662	1.39	.003	.179	.227	.213	.22
664	.80	.002	.057	.060	.035	.04	664	1.07	.003	.123	.132	.068	.07	664	1.24	.002	.173	.213	.182	.18
933	-.02	.000	-.001	-.002	.000	.00	933	-.32	.000	-.069	-.072	-.032	-.03	933	-.54	.000	-.130	-.156	-.122	-.12

37 RB +1 9 F -1		A = 5.63 DT = 238 T = 20 U2 = .0034 .0054 .0147	RBF		37 RB +1 17 CL -1		A = 6.56 DT = 176 T = 20 U2 = .0040 .0079 .0230	RBCL		37 RB +1 35 RB -1		A = 6.86 DT = 128 T = 20 U2 = .0042 .0105 .0316	RBBR							
G	VG	CIG(EL)		CIG(PHONON)	VIG	G	VG	CIG(EL)		CIG(PHONON)	VIG	G	VG	CIG(EL)		CIG(PHONON)	VIG			
000	12.19	1.506	.210	.270	.434	1.94	000	9.78	1.471	.174	.249	.402	1.87	000	9.85	1.606	.244	.388	.608	2.21
111	2.81	.039	.169	.214	.329	.37	111	.25	.019	.088	.121	.175	.19	111	-.86	-.002	-.009	-.018	-.037	-.04
200	6.04	.080	.206	.263	.414	.49	200	5.53	.074	.169	.242	.380	.45	200	5.94	.086	.239	.377	.572	.66
220	4.93	.039	.201	.255	.390	.43	220	4.54	.035	.170	.239	.361	.40	220	4.90	.044	.241	.372	.538	.58
311	1.83	.009	.159	.199	.290	.30	311	.43	.010	.085	.115	.158	.17	311	-.34	.001	-.008	-.016	-.028	-.03
222	4.16	.024	.203	.255	.375	.40	222	3.84	.020	.166	.231	.337	.36	222	4.18	.028	.230	.354	.493	.52
400	3.61	.017	.194	.242	.347	.36	400	3.32	.014	.162	.223	.314	.33	400	3.65	.020	.227	.343	.457	.48
331	1.36	.006	.155	.191	.260	.27	331	.45	.006	.085	.113	.145	.15	331	-1.17	.000	-.007	-.013	-.021	-.02
422	2.86	.011	.186	.228	.307	.32	422	2.62	.009	.154	.208	.272	.28	422	2.93	.013	.220	.322	.391	.40
440	2.38	.008	.177	.214	.270	.28	440	2.17	.007	.149	.197	.238	.24	440	2.47	.010	.212	.301	.334	.34
620	2.05	.006	.169	.202	.239	.25	620	1.86	.006	.143	.185	.207	.21	620	2.15	.009	.207	.284	.287	.30
444	1.80	.005	.162	.190	.211	.22	444	1.63	.005	.138	.174	.181	.19	444	1.91	.007	.198	.265	.244	.25
642	1.62	.004	.156	.180	.187	.19	642	1.46	.005	.134	.165	.159	.16	642	1.72	.006	.193	.249	.208	.21
733	.64	.003	.121	.136	.128	.13	733	.32	.002	.070	.083	.072	.07	733	-.05	.000	-.004	-.006	-.005	.00
662	1.30	.003	.139	.156	.137	.14	662	1.17	.004	.122	.143	.113	.12	662	1.41	.005	.177	.212	.139	.14
664	1.16	.002	.132	.143	.114	.12	664	1.05	.003	.115	.131	.092	.10	664	1.28	.004	.168	.193	.109	.11
933	.51	.001	.105	.112	.081	.08	933	.27	.000	.062	.068	.044	.05	933	-.03	.000	-.003	-.004	-.002	.00

37 RB +1 53 I -1		A = 7.32 DT = 108 T = 20 U2 = .0040 .0113 .0344	RBI		47 AG +1 17 CL -1		A = 5.55 DT = 136 T = 20 U2 = .0045 .0108 .0324	AGCL		47 AG +1 35 RB -1		A = 5.76 DT = 140 T = 20 U2 = .0033 .0078 .0233	AGBR							
G	VG	CIG(EL)		CIG(PHONON)	VIG	G	VG	CIG(EL)		CIG(PHONON)	VIG	G	VG	CIG(EL)		CIG(PHONON)	VIG			
000	9.95	1.822	.306	.516	.813	2.64	000	14.32	2.515	.438	.681	1.070	3.59	000	15.01	2.792	.463	.726	1.169	3.96
111	-2.24	-.022	-.119	-.202	-.321	-.34	111	1.23	.055	.287	.425	.609	.66	111	-.59	.025	.086	.127	.187	.21
200	6.41	.092	.302	.505	.769	.86	200	9.18	.109	.426	.653	.973	1.08	200	9.89	.127	.454	.705	1.095	1.22
220	5.27	.048	.308	.501	.728	.78	220	7.33	.050	.417	.624	.870	.92	220	7.94	.063	.443	.679	1.009	1.07
311	-1.23	-.004	-.112	-.186	-.273	-.28	311	1.52	.021	.269	.388	.497	.52	311	.32	.009	.085	.123	.171	.18
222	4.51	.031	.293	.475	.668	.70	222	6.13	.029	.414	.601	.778	.81	222	6.68	.039	.433	.652	.927	.97
400	3.95	.023	.290	.462	.621	.64	400	5.26	.019	.393	.560	.680	.70	400	5.80	.027	.430	.635	.856	.88
331	-.86	-.003	-.107	-.172	-.231	-.23	331	1.41	.009	.261	.358	.402	.41	331	.46	.001	.083	.117	.149	.15
422	3.20	.015	.283	.436	.537	.55	422	4.12	.010	.371	.504	.528	.54	422	4.62	.016	.406	.583	.715	.73
440	2.71	.011	.273	.409	.461	.47	440	3.38	.007	.349	.452	.408	.42	440	3.86	.012	.391	.542	.601	.61
620	2.37	.009	.264	.383	.395	.40	620	2.88	.006	.332	.409	.316	.32	620	3.33	.009	.376	.504	.506	.51
444	2.11	.007	.256	.360	.339	.35	444	2.51	.005	.312	.367	.243	.25	444	2.94	.007	.360	.468	.423	.43
642	1.92	.006	.247	.337	.290	.30	642	2.23	.005	.296	.331	.186	.19	642	2.64	.006	.345	.434	.354	.36
733	-.37	.000	-.088	-.115	-.088	-.09	733	.73	.004	.191	.199	.091	.09	733	.21	.000	.066	.076	.052	.05
662	1.56	.004	.230	.290	.197	.20	662	1.76	.004	.200	.258	.093	.10	662	2.11	.004	.313	.363	.226	.23
664	1.42	.004	.220	.265	.156	.16	664	1.56	.004	.242	.223	.059	.06	664	1.89	.004	.295	.326	.172	.18
933	-.28	.000	-.078	-.089	-.045	-.04	933	.54	.001	.158	.136	.029	.03	933	.13	.001	.057	.058	.025	.03

cally as a function of Z . This feature is remarkable since $C_{0g}^i(\text{el})$ is a function of the inelastic atomic scattering factors, while V_g is proportional to the elastic scattering factors. Usually the ratio of total inelastic over elastic scattering cross sections decreases with the atomic number Z (Lenz, 1954).

As to the absolute values, $C_{0g}^i(\text{el})$ ($g \neq 0$) gives only a small contribution to V_g^i at normal temperatures. It is more important for the lighter elements than for the

heavy ones and it becomes more important at low temperatures where the phonon scattering is fading out.

APPENDIX

Evaluation of the integral

The integral

$$\mathcal{I} = \int_0^{2\pi} \frac{B(d \cos \varphi)}{c^2 - d^2 \cos^2 \varphi} d\varphi$$

Table 2 (cont.)

55 CS +1						12 MG +0						12 MG +2								
9 F -1						8 O +0						8 O -2								
A= 6.01						A= 4.21						A= 4.21								
DT= 184						DT= 820						DT= 820								
T= 20						T= 20						T= 20								
U2=.0030 .0058 .0168						U2=.0025 .0026 .0038						U2=.0025 .0026 .0038								
G	VG	CIG(EL)	CIG(PHONON)	VIG		G	VG	CIG(EL)	CIG(PHONON)	VIG		G	VG	CIG(EL)	CIG(PHONON)	VIG				
000	13.55	1.904	.329	.467	.763	2.67	000	16.23	4.967	.081	.082	.104	5.07	000	17.94	1.703	.077	.078	.099	1.80
111	4.66	.061	.291	.409	.652	.71	111	1.26	-.002	.027	.027	.033	.03	111	-2.78	-.008	-.012	-.012	-.017	-.03
200	7.21	.093	.325	.458	.731	.82	200	7.11	.054	.077	.078	.099	.15	200	5.95	.066	.074	.075	.095	.16
220	5.85	.045	.318	.444	.688	.73	220	4.95	.026	.074	.075	.094	.12	220	4.59	.030	.072	.072	.091	.12
311	2.46	.014	.284	.391	.586	.60	311	.32	.007	.025	.025	.031	.04	311	-1.09	.005	-.009	-.009	-.012	.00
222	4.93	.029	.311	.430	.647	.68	222	3.90	.016	.070	.071	.088	.10	222	3.76	.017	.068	.069	.086	.10
400	4.28	.021	.311	.425	.615	.64	400	3.25	.010	.068	.068	.084	.09	400	3.18	.011	.065	.066	.081	.09
331	2.20	.009	.273	.369	.516	.53	331	.31	.004	.024	.024	.029	.03	331	-2.55	.003	-.006	-.006	-.008	.00
422	3.41	.013	.295	.396	.539	.55	422	2.46	.005	.062	.062	.075	.08	422	2.44	.005	.060	.060	.073	.08
440	2.86	.009	.286	.376	.479	.49	440	1.98	.003	.057	.057	.068	.07	440	1.98	.003	.055	.055	.066	.07
620	2.48	.007	.274	.354	.422	.43	620	1.66	.002	.053	.053	.062	.06	620	1.66	.002	.051	.051	.059	.06
444	2.19	.005	.266	.337	.374	.38	444	1.43	.001	.049	.049	.056	.06	444	1.43	.001	.047	.047	.054	.06
642	1.97	.004	.254	.315	.329	.33	642	1.25	.001	.046	.046	.051	.05	642	1.25	.001	.044	.044	.049	.05
733	1.02	.003	.221	.267	.252	.25	733	.21	.000	.016	.016	.018	.02	733	-.05	.000	.000	.000	.000	.00
662	1.58	.003	.233	.276	.242	.24	662	.96	.000	.038	.039	.041	.04	662	.96	.000	.037	.037	.040	.04
664	1.41	.002	.223	.257	.202	.20	664	.84	.000	.035	.035	.036	.04	664	.84	.000	.034	.034	.035	.04
933	.78	.001	.195	.218	.156	.16	933	.15	.000	.013	.013	.013	.01	933	-.02	.000	.000	.000	.000	.00

82 PB +2						82 PB +2						82 PR +2								
16 S -2						34 SE -2						52 TE -2								
A= 5.94						A= 6.14						A= 6.44								
DT= 230						DT= 168						DT= 139								
T= 20						T= 20						T= 20								
U2=.0015 .0025 .0069						U2=.0018 .0036 .0107						U2=.0019 .0044 .0133								
G	VG	CIG(EL)	CIG(PHONON)	VIG		G	VG	CIG(EL)	CIG(PHONON)	VIG		G	VG	CIG(EL)	CIG(PHONON)	VIG				
000	16.88	2.976	.466	.614	1.014	3.99	000	17.11	3.169	.537	.797	1.312	4.48	000	17.74	3.588	.586	.936	1.544	5.13
111	2.19	.042	.393	.512	.824	.91	111	.58	.052	.314	.450	.696	.75	111	-1.64	.026	.177	.259	.362	.39
200	9.95	.128	.462	.607	.994	1.12	200	10.44	.143	.536	.791	1.283	1.43	200	11.15	.150	.591	.937	1.513	1.66
220	7.94	.065	.456	.597	.965	1.03	220	8.39	.077	.528	.775	1.235	1.31	220	8.92	.081	.582	.916	1.450	1.53
311	2.31	.036	.393	.509	.799	.84	311	1.19	.024	.315	.448	.670	.69	311	-.18	.017	.181	.263	.360	.38
222	6.68	.042	.448	.585	.935	.98	222	7.11	.051	.519	.759	1.188	1.24	222	7.55	.054	.572	.895	1.389	1.44
400	5.81	.031	.455	.591	.926	.96	400	6.22	.038	.527	.764	1.164	1.20	400	6.61	.041	.582	.901	1.355	1.40
331	2.15	.020	.384	.496	.759	.78	331	1.23	.013	.309	.435	.629	.64	331	.19	.009	.179	.259	.345	.35
422	4.65	.019	.437	.565	.863	.88	422	5.05	.025	.506	.726	1.068	1.09	422	5.39	.026	.550	.841	1.222	1.25
440	3.42	.014	.432	.553	.821	.83	440	4.30	.018	.500	.707	.997	1.01	440	4.60	.019	.540	.813	1.126	1.14
620	3.40	.011	.420	.534	.771	.78	620	3.77	.014	.480	.672	.914	.93	620	4.04	.014	.526	.779	1.032	1.05
444	3.02	.009	.414	.523	.733	.74	444	3.37	.011	.471	.651	.849	.86	444	3.61	.011	.514	.749	.948	.96
642	2.72	.008	.406	.509	.694	.70	642	3.05	.009	.462	.630	.789	.80	642	3.27	.009	.507	.728	.877	.89
733	1.27	.004	.344	.427	.555	.56	733	.76	.002	.278	.367	.422	.42	733	.30	.002	.165	.223	.240	.24
662	2.20	.006	.383	.473	.601	.61	662	2.49	.007	.439	.581	.657	.66	662	2.66	.006	.480	.663	.711	.72
664	1.98	.005	.375	.458	.558	.56	664	2.26	.005	.424	.551	.586	.59	664	2.40	.005	.463	.625	.626	.63
933	.99	.003	.320	.386	.450	.45	933	.59	.002	.257	.324	.320	.32	933	.26	.002	.155	.199	.183	.18

Table 3. Conversion factors

Conversion of absorption potentials for primary electron energies other than 100 keV is done by multiplying the tabulated values by the factor $v(100 \text{ keV})/v(E)$. (v is the velocity of an electron of energy E .)
 In order to obtain the frequently used absorption coefficients, $\mu_0 = -2m/(\hbar^2 k)$. C_{00}^i and $\Delta\mu_g = -2m/(\hbar^2 k)$. C_{0g}^i , the Table also contains the factor by which the $C^i(100 \text{ keV})$ -values of Tables 1 and 2 must be multiplied to give the absorption coefficients at the required energy E .

E (keV)	$\frac{v(100\text{keV})}{v(E)}$	$\frac{\mu(E)}{C^i(100 \text{ keV})}$ ($10^5 \text{ cm}^{-1}\text{eV}^{-1}$)
20	2.017	7.511
30	1.669	5.148
40	1.466	3.968
50	1.328	3.260
60	1.229	2.788
70	1.152	2.452
80	1.091	2.199
90	1.041	2.004
100	1.000	1.847
200	0.788	1.148
300	0.706	0.921
400	0.662	0.810
500	0.635	0.746

can be calculated in the closed form. Supposing $B(d \cos \varphi) = B(-d \cos \varphi)$, which is true for our applica-

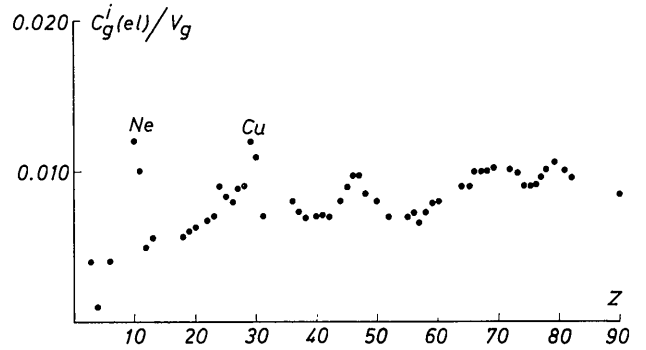


Fig. 2. $C_{0g}^i(El)/V_g$ for the fixed value $|g| = 3$. plotted versus Z .

tion, the integral becomes

$$\varphi = \frac{1}{c} \int_0^{2\pi} \frac{B(d \cos \varphi)}{c + d \cos \varphi} d\varphi.$$

On introduction of the complex variable

$$z = \exp i\varphi; \cos \varphi = (z + \bar{z})/2,$$

the integral is written as

$$\begin{aligned} \mathcal{J} &= \frac{1}{ic} \oint \frac{B[d(z+\bar{z})/2]dz}{z[c+d(z+\bar{z})/2]} \\ &= \frac{2}{icd} \oint \frac{B[d(z+\bar{z})/2]dz}{z^2+2(c/d)z+1} \end{aligned}$$

Integration over the unit circle is carried out by means of the residue theorem. The zeros of the denominator

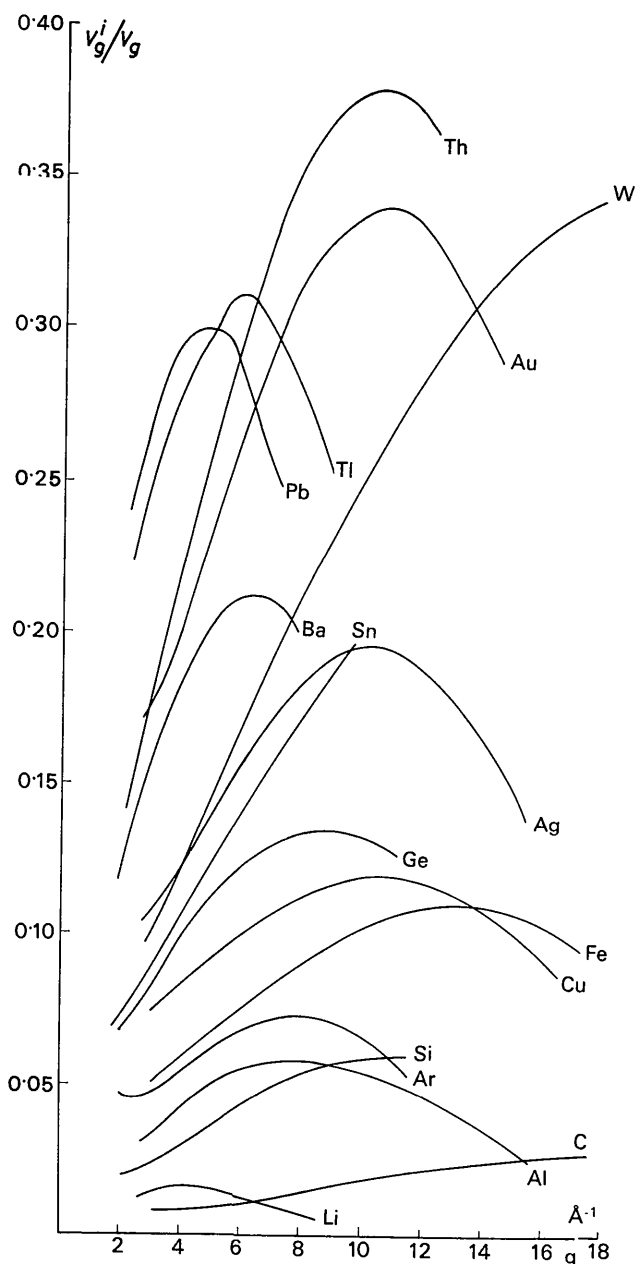


Fig. 3. V_g^i/V_g as a function of g for a number of monatomic crystals. The curves start at the lowest reflection g . The values at $g=0$ are not drawn because the curves would overlap here, since C_{00}^i (plasmon) makes a δ -shaped contribution to V_g^i at $g=0$.

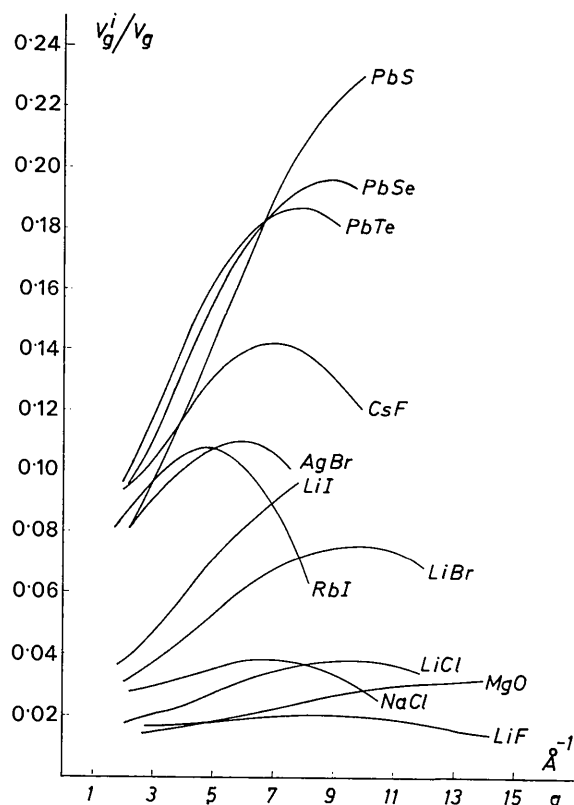


Fig. 4. As Fig. 3, but showing compounds of the NaCl-type.

occur at

$$z_{\pm} = -\frac{c}{d} \pm \sqrt{c^2/d^2 - 1}$$

assuming $c \geq d$. Of the two poles of the integrand only z_+ lies within the unit circle (on the real axis). This gives

$$\mathcal{J} = \frac{4\pi}{cd} \frac{B(dz_+)}{2z_+ + 2c/d} = \frac{2\pi}{c} \frac{B(-c + \sqrt{c^2 - d^2})}{\sqrt{c^2 - d^2}}$$

In our application

$$c = a^2 + r^2$$

$$d = 2ar \quad (c \geq d)$$

$$\sqrt{c^2 - d^2} = |a^2 - r^2|,$$

then

$$\mathcal{J} = \frac{2\pi B[-(a^2 + r^2) + |a^2 - r^2|]}{(a^2 + r^2) |a^2 - r^2|}.$$

The computations were carried out on the CDC 3300 at the Zentrum für Datenverarbeitung der Universität Tübingen.

The temperature dependence of V_g and $C_{0g}^i(\text{el})$ can be described in a satisfactory way (exact for a rigid-ion Einstein model) by multiplying the required potential

Table 4. Comparison of theoretical and experimental values of V_g^i at 300°K

The data refer to 100 keV electrons. Some of the experimental values have been extrapolated to 100 keV.

	g	V_g^i	V_g^i	References
		theor. (eV)	exp. (eV)	
MgO	000	1.80	1.50	Goodman & Lehmpfuhl (1967).
	200	0.16	{0.13 0.55	
Al	000	0.85	0.50	Hashimoto (1964). See also Metherell & Whelan (1967) and Watanabe (1965).
	111	0.18	0.23	
Cu	000	3.48	1.35	Goringe & Whelan (1966).
	111	0.83	0.81	
	200	0.79	0.92	
Si	000	0.70	0.68	Meyer-Ehmsen (1969).
	220	0.11	0.11	
	311	0.07	0.08	
	422	0.08	0.08	
Ge	000	1.56	1.25	Meyer-Ehmsen (1969).
	220	0.54	0.52	
	422	0.43	0.36	
NaCl	000	1.63		Graff & Gaukler (1969).
	220	0.20	0.21	
	420	0.14	0.15	

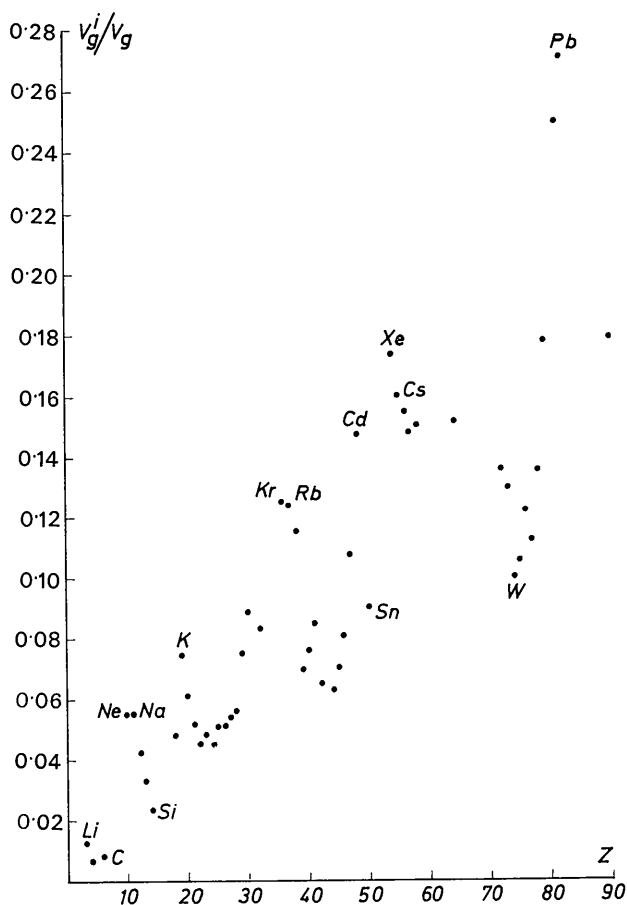


Fig. 5. V_g^i/V_g for the fixed value $|g| = 3$ plotted versus Z .

with the corresponding Debye-Waller factor, $\exp[-M(g)]$, as shown by Ohtsuki (1966).

The main contribution to V_g^i is from C_{0g}^i (phonon).

Because of the properties of $C_{0g}^i(\text{el})/V_g$, the curves V_g^i/V_g given in Figs. 3, 4 and 5 can approximately be taken to be qualitatively the same as that for C_{0g}^i (phonon)/ V_g or, with an appropriate factor, $C_{0g}^i(\text{phonon})/C_{0g}^i(\text{el})$. The values of $C_{0g}^i(\text{phonon})/V_g$ increase with g , with atomic weight and with temperature (see also Whelan, 1965*b* and Humphreys & Hirsch, 1968).

For a given crystal (Figs. 4 and 5), V_g^i/V_g increases approximately linearly with g (in the region of practical interest), goes through a maximum and falls again (to negative values at very high g).

For a given g , the absorption parameter V_g^i/V_g increases in a rough linear way with the atomic weight, and therefore also with Z (see Fig. 5). The form of the plot is not as simple to explain as that of atomic radii, since the phonon scattering depends on the Debye temperature and therefore on structure effects. Crystals of the diamond type, for instance, are on the lower fringe, whereas alkali and rare gas crystals appear on the upper fringe of the plot. Nevertheless, a variation with position in the periodic system is evident.

The parameter V_g^i/V_0 (see Fig. 6 for some examples) is a measure of the 'localization' of the inelastic scattering probability at the atomic planes. Phonon scattering is strongly, and electron excitation weakly, localized (see also Whelan, 1965*a,b*). The parameter V_g^i/V_0 determines the strength of the anomalous transmission effects.

A large part of V_0^i consists of the plasmon contribution, which is shown separately in Tables 1 and 2. It is not meant to represent an actual plasmon excitation of energy, EP, in every case, but rather to indicate the contribution, $C_{00}^i(\text{plasmon})$, of the 'free' electrons to $C_{0g}^i(\text{el})$ which also contains the contribution from single electron excitations.

In Table 2 the crystal MgO is listed twice, with different assumptions about the electron distributions within the crystal.

In Table 4 theoretical values in Tables 1 and 2 are compared with experimental values of V_g^i . Agreement is satisfactory in most cases. Exceptions at V_0^i may be due to theoretical assumptions about the minimum scattering wave vector while slight deviations at V_g^i may be partly explained on the experimental side, where evaluation is rendered difficult by complications such as, for instance, many-beam effects.

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The Dynamical Scattering Amplitude of an Imperfect Crystal

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A formal expression for the X-ray dynamical scattering amplitude of an imperfect crystal is obtained. The resultant expression includes the dynamical line broadening effect on the diffracted X-rays caused by imperfections. The effects due to absorption in the crystal are also taken into account.

1. Introduction

In studying the problems of diffraction by crystals it is desirable to have a dynamical solution for diffracted beams because, even under a single Bragg condition, there are strong interactions between the crystal electrons and the beams. When the crystal is perfect, one uses the two-wave approximation to derive an approximate dynamical solution for a single Bragg reflection. When an incident beam satisfies several Bragg conditions simultaneously, one needs to employ 'more than two'-wave approximations which are sometimes only solvable by numerical computations.

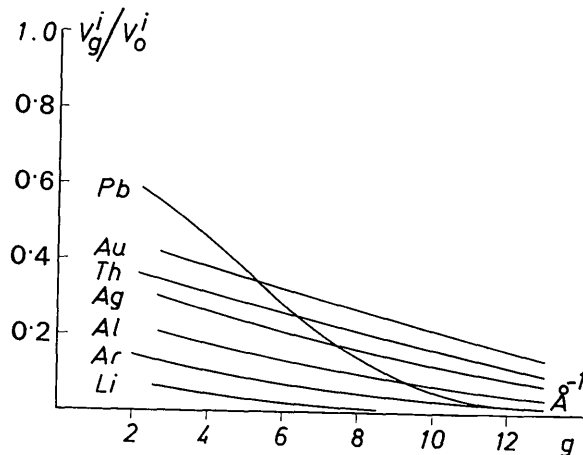


Fig. 6. V_g^i/V_0^i as a function of g for some crystals.

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There is no true dynamical theory for diffraction unless the multiple interaction is correctly treated. Since difficulty is encountered in obtaining dynamical solutions for a perfect crystal, greater difficulty is expected in seeking completely dynamical solutions for an imperfect crystal.

In electron diffraction where the multiple scattering is essential, a formal theory of dynamical diffraction has been formulated by Niehrs (1959a, b) and Fujimoto (1959, 1960). In this theory the scattering amplitude is completely dynamical in form. In practice, it is admittedly tedious to calculate the scattering amplitude to a great degree of accuracy. Nevertheless this am-